| https://lh4.googleusercontent.com/SPvkLsQz-5KaF-7uqXwqhrjwmbNxZ4qCCZAnhg3-Q024TrYF0A_9l6QyNZ5Zvzu3aH8itLneRVEpMIqsV1S89vn2YxwuiAoTc1TlDXvGevlf1TAttRmsGETDwmeqShyvP3EjOaiE | **PES University, Bengaluru**  (Established under Karnataka Act No. 16 of 2013) | **UE20CS904** |
| --- | --- | --- |
| **Jan24 : END SEMESTER ASSESSMENT (ESA)**  **M TECH DATA SCIENCE AND MACHINE LEARNING\_ SEMESTER I**  **UE20CS904 - Mathematical Foundation** | | |
| Time: 3 Hrs | Answer All Questions | Max Marks: 100 |
| **Instructions**   * Section A all answers should be handwritten in the answer script. * Section B and Section C has to be answered in Jupyter notebook (python code) | | |
| **Section A (20 marks)** | | |

| 1 | a) | Define the concept of a unit vector and explain its significance in vector operations.  Calculate the angle between two given vectors. The two vectors are,  a = + 2 and  b = 9 + 3 | 4 |
| --- | --- | --- | --- |
| b) | Write down all the arithmetic operations possible on vectors.  Find the vector projection of the vector  on | 4 |
| c) | What are local minima and global minima ?  Find out the minima of the following function for the interval ( -5, -2)  f(x)=x3+2x | 4 |
| d) | Write the transformation matrix for rotation and reflection of a 2d image. | 4 |
|  | e) | What are critical points in a function? Find the critical points of the function f(x)=x5−5x4+5x3−1 | 4 |
| **Section B (40 marks)** | | | |
| 2 | a) | Consider the following matrix and answer the below 2 questions  A=[3,1,2] , B=[-1,3,4 ], C=[5, 0, 1]  (i) Check whether the vectors are linearly independent and find the rank of the matrix. (4 marks)  (ii) Check if this matrix is orthogonal. (3 marks) | 7 |
| b) | Find Eigen Values of and for ,  A=[4 3 2 1] | 7 |
|  | c) | Find the scalar and vector projections of the vector u = 8i + 6j on vector v = i - 7j | 6 |
|  | d) | Find singular value decomposition of  A= | 6 |
|  | e) | Convert the following matrix to an orthogonal matrix using Gram Schmidt Process?  A=[1 1 1];[-1 0 1];[1 1 2] | 6 |
|  | f) | A=[1 1 1 1] B=[0 1 −1 1] C=[−2 1 0 0] Check if the following are true or false:   1. (BA + A) = (B + I)A, where I is the identity matrix. 2. (A + B)C = AC + BC | 8 |
|  | | | |
|  |  | **SECTION C – (40 MARKS)** |  |
| 4 | a) | consider the image “Lena.jpg” and follow the below mentioned operations.   1. Load the image and display 2. Apply grayscale transformation 3. Apply Gaussian blur filter 4. Apply edge detection (Sobel filter) 5. Rotate the image by 45 degrees 6. Scale the image 7. Translate the image | 10 |
| 4 | b) | Apply PCA step by step on Iris dataset and find out the principal components that capture 95% variances. (follow the steps given in the notebook) | 15 |
| 4 | c) | Consider the data given below and fit a linear regression line using gradient descent.   |  |  |  | 0.6 | 1 | | --- | --- | --- | --- | --- | |  |  |  | 0.48 | 0.95 |   Initialize the weights a and b to 0.8, 0.2 respectively. Update the weights such that the error is minimum using gradient descent. Use the function sum of squared errorswhere is the y-predicted value and is the actual given y. Plot the linear regression line after updating the values of a and b in two iterations. | 15 |